

## Warm-Up!

The manager of 3 vacation condo resorts wants to see how much money he can take in by renting all of his condos for one week. 2-bedrooms rent for \$1700 for the week, 3-bedrooms for \$2000 and 4-bedrooms for \$2500. Write a matrix to represent the number of each type of condo at each resort and a matrix for the rental cost for each. Then calculate the total rent generated in one week, at each condo, if everything is rented.

	2-bedroom	3-bedroom	4-bedroom
Sunny Side	30	25	20
Beach Comber	15	40	10
Surfs Up	40	45	15

$$\begin{bmatrix} 30 & 25 & 20 \\ 15 & 40 & 10 \\ 40 & 45 & 15 \end{bmatrix} \begin{bmatrix} 1700 \\ 2000 \\ 2500 \end{bmatrix}$$

## 4-5 Determinants

The determinant of a matrix is a value of a square matrix that will be used to:

- calculate the inverse of a matrix
- solve systems

• area of polygons

2x2 Matrix (2nd order determinants)  
The determinant, D, is calculated as follows:

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

ex:

$$\begin{vmatrix} 5 & 10 \\ 8 & -3 \end{vmatrix} = -15 - 80 = -95$$

matrix  
calculate determ.

ex:

$$\begin{vmatrix} 2 & 6 \\ -5 & 1 \end{vmatrix} = -2 - -30 = 28$$

ex:

$$\begin{vmatrix} 4 & 10 \\ 2 & 5 \end{vmatrix} = 20 - 20 = 0$$

ex:

$$\begin{vmatrix} 4 & 6 \\ x & 12 \end{vmatrix} = 12$$

$$48 - 6x = 12$$

$$-6x = -36$$

$$x = 6$$

3x3 Matrix (3rd order determinants)  
Before calculating the determinant, D, we must define a minor.

Minor--of an element in a determinant is the determinant resulting from the deletion of the row and column containing the element.

ex:

$$\begin{vmatrix} 5 & -1 & -2 \\ 3 & 6 & -7 \\ 2 & -3 & 4 \end{vmatrix}$$

The minor of 5 is:

$$\begin{vmatrix} 6 & -7 \\ -3 & 4 \end{vmatrix}$$

The minor of 6 is:

$$\begin{vmatrix} 5 & -2 \\ 2 & 4 \end{vmatrix}$$

The minor of -7 is:

$$\begin{vmatrix} 5 & -1 \\ 2 & -3 \end{vmatrix}$$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 5 & -1 & -2 \\ 3 & 6 & -7 \\ 2 & -3 & 4 \end{vmatrix} = 83$$

$$5 \begin{vmatrix} 6 & -7 \\ -3 & 4 \end{vmatrix} + 1 \begin{vmatrix} 3 & -7 \\ 2 & 4 \end{vmatrix} + 2 \begin{vmatrix} 3 & 6 \\ 2 & -3 \end{vmatrix}$$

$$5 \begin{vmatrix} 6 & -7 \\ -3 & 4 \end{vmatrix} + 1 \begin{vmatrix} 3 & -7 \\ 2 & 4 \end{vmatrix} + 2 \begin{vmatrix} 3 & 6 \\ 2 & -3 \end{vmatrix}$$

$$15 + 26 + 42 = 83$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 2 & -1 & -6 \\ 3 & 4 & 2 \\ 5 & -2 & 3 \end{vmatrix}$$

$$-1 \begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} + 4 \begin{vmatrix} 2 & -6 \\ 5 & 3 \end{vmatrix} - 2 \begin{vmatrix} 2 & -6 \\ 3 & 2 \end{vmatrix}$$

$$-1 \begin{vmatrix} 3 & 2 \\ 5 & 3 \end{vmatrix} + 4 \begin{vmatrix} 2 & -6 \\ 5 & 3 \end{vmatrix} - 2 \begin{vmatrix} 2 & -6 \\ 3 & 2 \end{vmatrix}$$

$$-1(-1) + 4(-30) - 2(-18)$$

$$1 - 120 + 36 = -83$$

Evaluate the determinant, using expansion by minors.

ex:

$$\begin{vmatrix} 5 & 2 & 34 \\ -1 & 3 & 22 \\ 0 & 0 & 4 \end{vmatrix}$$

$$+0 \begin{vmatrix} 1 & -0 \\ 1 & -0 \end{vmatrix} + 4 \begin{vmatrix} 5 & 2 \\ -1 & 3 \end{vmatrix}$$

$$17 + 15 = 68$$

Expansion by minors works for any higher order determinant as well.

ex:

$$\begin{vmatrix} 1 & 2 & 1 & 0 \\ 3 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{vmatrix}$$

hw

p186 15-19 odd, 27-30, 39